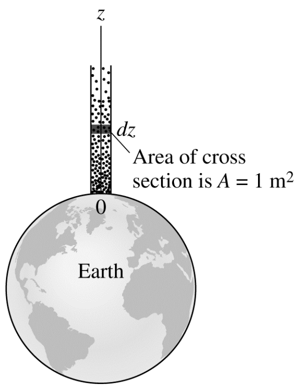
15.45. Visualize:



The figure shows a small column of air of thickness dz, of cross-sectional area $A = 1\ \text{m}^2$, and of density $\rho(z)$. The column is at a height z above the surface of the earth.

Solve: **(a)** The atmospheric pressure at sea level is 1.013×10^5 Pa. That is, the weight of the air column with a $1\ \text{m}^2$ cross section is 1.013×10^5 N. Consider the weight of a $1\ \text{m}^2$ slice of thickness dz at a height z. This slice has volume $dV = Adz = (1\ \text{m}^2)dz$, so its weight is $dw = (\rho dV)g = \rho g(1\ \text{m}^2)dz = \rho_0 e^{-z/z_0} g(1\ \text{m}^2)dz$. The total weight of the $1\ \text{m}^2$ column is found by adding all the dw. Integrating from $z = 0$ to $z = \infty$,

$$\begin{aligned} w &= \int_0^\infty \rho_0 g(1\ \text{m}^2)e^{-z/z_0}dz \\ &= \left(-\rho_0 g(1\ \text{m}^2)z_0\right)\left[e^{-z/z_0}\right]_0^\infty \\ &= \rho_0 g(1\ \text{m}^2)z_0 \end{aligned}$$

Because $w = 101{,}300\ \text{N} = \rho_0 g(1\ \text{m}^2)z_0$,

$$z_0 = \frac{101{,}300\ \text{N}}{(1.28\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(1.0\ \text{m}^2)} = 8.08 \times 10^3\ \text{m}$$

(b) Using the density at sea level from Table 15.1,

$$\rho = (1.28\ \text{kg/m}^3)e^{-z/(8.08\times10^3\ \text{m})} = (1.28\ \text{kg/m}^3)e^{-1600\ \text{m}/(8.08\times10^3\ \text{m})} = 1.05\ \text{kg/m}^3$$

This is 82% of ρ_0.